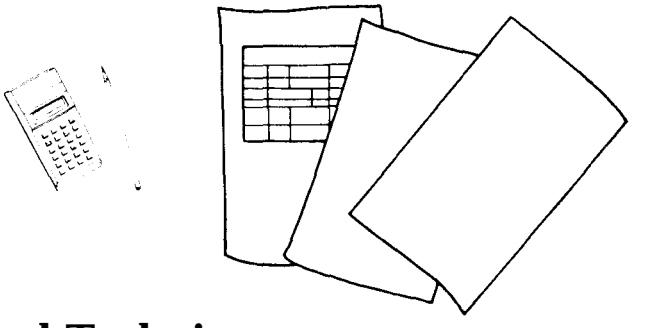


Mathematical Techniques

GEOMETRIC FUNCTIONS

The result of any operation performed by terrain analysts will only be as accurate as the measurements used. An interpretation and detailed analysis often requires terrain analysts to use basic geometric functions. In addition, approximation of volume, extent of stock piles, and capacity of floor space require geometric formulas. Therefore, analysts must have a sound background in the fundamentals of basic imagery math. This section serves as a guide to a unique and comprehensive collection of current geometric formulas that are basic to all types of imagery interpretation.

Determination of Area

1. Rectangles FORMULA: $A = l \times W$

Where-- A = Area
l = Length
W = Width

2. Circles FORMULA: $A = 2\pi \times r$ *or* $\pi \times d$

Where-- A = Area
d = Diameter
r = Radius
π = 3.142

3. Triangles FORMULA: $A = \frac{b \times h}{2}$

Where-- A = Area
b = Base of the triangle
h = Height of the triangle

4. General quadrilaterals FORMULA: $A = \frac{1}{2} b (h_1 + h_2)$ or $A = \frac{b (h_1 + h_2)}{2}$

Where-- A = Area
h_1 = Height no. 1
h_2 = Height no. 2
b = Base common to two triangles formed by the diagonal

Table 10-1 Area Formulas

5. Ellipses FORMULA: $A = \pi x a x b$

Where-- A = Area
π = 3.142
a = Semi-axis
b = Semi-axis

Table 10-1 Area Formulas - continued

Determination of Volume

1. Rectangular solids FORMULA: $V = l x w x h$

Where-- V = Volume
l = Length
w = Width
h = Height

2. Cylinders FORMULA: $V = \pi x r^2 x h$

Where-- V = Volume
π = 3.142
r = Radius
h = Height

3. Spheres FORMULA: $V = \frac{4}{3} \pi r^3$

Where-- V = Volume
π = 3.142
r = Radius
3 = Constant

4. Cones FORMULA: $V = \frac{\pi x r^2 x h}{3}$

Where-- V = Volume
π = 3.142
r = Radius
h = Height of cone
3 = Constant

5. Triangular solids FORMULA: $V = \frac{l x w x h}{2}$

Where-- V = Volume
l = Length
w = Width
h = Height
2 = Constant

Table 10-2 Volume Formulas

6. Trapezoidal solids FORMULA: $V = \frac{(l + l')\,h\,(w + w')}{2}$

Where--
V = Volume
l = Length of base
l' = Length of top
w = Width of base
w' = Width of top
h = Height
2 = Constant

7. **Irregular triangular solids** FORMULA: $V = \frac{w \times h \times (2a + c)}{6}$

Where--
V = Volume
w = Width
h = Height
a = Length of base
c = Length of peak
6 = Constant

Table 10-2 Volume Formulas - continued

Angle of Repose

The angle of repose of any material is the angle at which material will stand when piled. Moisture content is often the controlling factor. The percent of fine material in the mass has a decided influence on the angle, as the fine carries the bulk of the moisture. Screened material has an angle of repose of 35 to 40 degrees.

Table 10-1 gives the average angle of repose and the average weight per cubic foot of various materials of interest to the terrain analyst.

MATERIAL	ANGLE OF REPOSE (Degrees)	TAN	WEIGHT PER CUBIC FT.
sand, dry	27	0.50952	100 lbs
nd, wet	30	0.57735	115 lbs
avel	39	0.80978	117 lbs
earth, dry	35	0.70021	90 lbs
coal, anthracite	27	0.50952	52 lbs
coal, bituminous	32	0.62487	47 lbs
coke, industrial	40	0.83910	28 lbs
iron ore, crushed	37	0.75355	145 lbs
copper ore, crushed	37	0.75355	262 lbs
limestone, crushed	37	0.75355	165 lbs

Table 10-3. Angle of Repose

Thus, knowing the angle of repose of a specific material, we can solve for height, using the horizontal dimensions.

Example: Find the height (h) of a pile of limestone.

Formula: $h = Tan\ A\ (\frac{1}{2}b)$

h = 0.75355 x 20 ft.
h = 15.071 ft.

Where-- b = 40 feet
Angle A = 37 degrees
Tan 37° = 0.75355 (from Table 10-1)

TRIGONOMETRIC FUNCTIONS

Oblique photos and thermal and SLAR imagery are often used to supplement or take the place of vertical photographs. In order to ensure maximum use of this imagery, the interpreter must be familiar with certain concepts, formulas, and principles concerning their accurate interpretation. This section gives the terrain analyst a basic understanding of trigonometric functions necessary for solving problems dealing with these special types of imagery, as well as a table of trigonometric conversions.

Six Functions

To define the six trigonometric functions upon which trigonometry is based, consider the angle, initial side AQ, and terminal side AS.

Choose any point B, different from A on the terminal side AS, and drop a perpendicular BC to the initial side AQ. From the three sides (BC, AC, and AB) associated with point B, six ratios are formed which are called the six trigonometric functions of Q.

$$\frac{CB}{AB}, \frac{AC}{AB}, \frac{CB}{AC}, \frac{AB}{CB}, \frac{AB}{AC}, \text{ and } \frac{AC}{CB}$$

These ratios are independent of the position of the point B on the freed terminal side AS, for if we choose any other point B1 on AS and drop the perpendicular BlC1 to the initial side AQ, the two right triangles ABC and AB1C1 are similar. Therefore, their corresponding sides have the same ratios.

If the angle is placed in standard position on a coordinate system, and the distance AB is designated by c, the six trigonometric functions may be defined as follows:

$$\text{Sin angle} = \frac{CB}{AB} = \frac{Opp\ Side}{Hyp} = \frac{Side\ a}{Side\ b}$$

$$\text{Cos angle} = \frac{AC}{AB} = \frac{Adj\ Side}{Hyp} = \frac{Side\ b}{Side\ c}$$

$$\text{Tan angle} = \frac{CB}{AC} = \frac{Opp\ Side}{Adj\ Side} = \frac{Side\ a}{Side\ b}$$

$$\text{Cot angle} = \frac{AC}{CB} = \frac{Adj\ Side}{Opp\ Side} = \frac{Side\ b}{Side\ a}$$

$$\text{Sec angle} = \frac{AB}{AC} = \frac{Hyp}{Adj\ Side} = \frac{Side\ c}{Side\ b}$$

$$\text{Csc angle} = \frac{AB}{CB} = \frac{Hyp}{Adj\ Side} = \frac{Side\ c}{Side\ a}$$

To define trigonometric functions of the sides of right triangles, one will often find it convenient to use the functions of an acute angle. Thus, functions of the acute angle B may be written as follows:

$$\text{Sin angle B} = \frac{AC}{AB} = \frac{Opp\ Side}{Hyp} = \frac{Side\ b}{Side\ c}$$

$$\text{Cos angle B} = \frac{BC}{AB} = \frac{Adj\ Side}{Hyp} = \frac{Side\ a}{Side\ c}$$

$$\text{Tan angle B} = \frac{AC}{BC} = \frac{Opp\ Side}{Adj\ Side} = \frac{Side\ b}{Side\ a}$$

$$\text{Cot angle B} = \frac{BC}{AC} = \frac{Adj\ Side}{Opp\ Side} = \frac{Side\ a}{Side\ b}$$

$$\text{Sec angle B} = \frac{AB}{BC} = \frac{Hyp}{Adj\ Side} = \frac{Side\ c}{Side\ a}$$

$$\text{Csc angle B} = \frac{AB}{AC} = \frac{Hyp}{Adj\ Side} = \frac{Side\ c}{Side\ b}$$

By comparing the values of the functions of angle A and angle B, we find the following equations:

$$Sin\ A = Cos\ B \quad Tan\ A = Cot\ B \quad Sec\ A = Csc\ B$$
$$Cos\ A = Sin\ B \quad Cot\ A = Tan\ B \quad Csc\ A = Sec\ B$$

Since angles A and B are complementary, the cosine, cotangent, and cosecant are called cofunctions of the sine, tangent and secant respectively. Conversely, we may state the following theorem: Any trigonometric function of an acute angle is equal to the corresponding cofunction of its complementary angle.

We can use this theorem to express any function of an angle greater than 45 degrees in terms of a function of an angle less than 45 degrees. For this reason, tables of values of the trigonometric functions need be computed only for angles from 0 to 45 degrees instead of from 0 to 90 degrees.

Illustrations: (a) Sin 76°43' = Cos 90° -76° 43'= Cos 13°17'
(b) Cot 51° 28'9" = Tan 90° - 51°28' 9"= Tan 38°31'51"

Right Triangles

Terrain analysts will use the functions of the right triangle most often for interpretation purposes. In any triangle, the sum of the interior angles is 180 degrees. If the triangle to be solved is a right triangle, one of the known parts in any case is the angle C; that is, C = 90 degrees, and A + B = 90 degrees. To solve a right triangle, therefore, we must find two sides or one side and an acute angle, using the formulas--

A + B =90 degrees

$$Sin\ angle\ A = \frac{Opp\ Side}{Hyp} = \frac{a}{c}$$

$$Cos\ angle\ A = \frac{Adj\ Side}{Hyp} = \frac{b}{c}$$

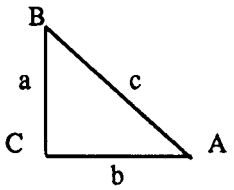
$$Tan\ angle\ A = \frac{Opp\ Side}{Adj\ Side} = \frac{a}{b}$$

If two sides are given, we can find the sin of angle A from the above formulas involving the two given sides. We may then use another formula to find the remaining side, and then find angle B by subtracting angle A from 90 degrees.

If one side of an acute angle is given, we begin by finding the other acute angle. Then we select one of the trigonometric formulas containing an unknown side and solve for it.

Pythagorean Theorem

The Pythagorean theorem provides a method for finding the lengths of the sides of a right triangle and checking the trigonometric method, with the statement: The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. Related to a right triangle labeled A, B, and C, as shown previously and reproduced here for convenience, the theorem may be stated as the formula



$$a^2 + b^2 = c^2$$

or

$$c^2 = a^2 + b^2$$

Note: In order to solve the formula, we must understand the solution to the square root function. The square root of a number is the result of a number multiplied by itself. For example, 2 x 2 = 4, so the square root of 4 is 2.

Degrees	Sines							Cosines
	0′	10′	20′	30′	40′	50′	60′	
0	0.00000	0.00291	0.00582	0.00873	0.01164	0.01454	0.01745	89
1	0.01745	0.02036	0.02327	0.02618	0.02908	0.03299	0.03490	88
2	0.03490	0.03781	0.04071	0.04362	0.04653	0.04943	0.05234	87
3	0.05234	0.05524	0.05814	0.06105	0.06395	0.06685	0.06976	86
4	0.06976	0.07266	0.07556	0.07846	0.08136	0.08426	0.08716	85
5	0.08716	0.09005	0.09295	0.09585	0.09874	0.10164	0.10453	84
6	0.10453	0.10742	0.11031	0.11320	0.11609	0.11898	0.12187	83
7	0.12187	0.12476	0.12764	0.13053	0.13341	0.13629	0.13917	82
8	0.13917	0.14205	0.14493	0.14781	0.15069	0.15356	0.15643	81
9	0.15643	0.15931	0.16218	0.16505	0.16792	0.17078	0.17365	80
10	0.17365	0.17651	0.17937	0.18224	0.18509	0.18795	0.19081	79
11	0.19081	0.19366	0.19652	0.19937	0.20222	0.20507	0.20791	78
12	0.20791	0.21076	0.21360	0.21644	0.21928	0.22212	0.22495	77
13	0.22495	0.22778	0.23062	0.23345	0.23627	0.23910	0.24192	76
14	0.24192	0.24474	0.24756	0.25038	0.25320	0.25601	0.25882	75
15	0.25882	0.26163	0.26443	0.26724	0.27004	0.27284	0.27564	74
16	0.27564	0.27843	0.28123	0.28402	0.28680	0.28959	0.29237	73
17	0.29237	0.29515	0.29793	0.30071	0.30348	0.30625	0.30902	72
18	0.30902	0.31178	0.31454	0.31730	0.32006	0.32282	0.32557	71
19	0.32557	0.32832	0.33106	0.33381	0.33655	0.33929	0.34202	70
20	0.34202	0.34475	0.34748	0.35021	0.35293	0.35565	0.35837	69
21	0.35837	0.36108	0.36379	0.36650	0.36921	0.37191	0.37461	68
22	0.37461	0.37730	0.37999	0.38268	0.38537	0.38805	0.39073	67
23	0.39073	0.39341	0.39608	0.39875	0.40142	0.40408	0.40674	66
24	0.40674	0.40939	0.41204	0.41469	0.41734	0.41998	0.42262	65
25	0.42262	0.42525	0.42788	0.43051	0.43313	0.43575	0.43837	64
26	0.43837	0.44098	0.44359	0.44620	0.44880	0.45140	0.45399	63
27	0.45399	0.45658	0.45917	0.46175	0.46433	0.46690	0.46947	62
28	0.46947	0.47204	0.47460	0.47716	0.47971	0.48226	0.48481	61
29	0.48481	0.48735	0.48989	0.49242	0.49495	0.49748	0.50000	60
30	0.50000	0.50252	0.50503	0.50754	0.51004	0.51254	0.51504	59
31	0.51504	0.51753	0.52002	0.52250	0.52498	0.52745	0.52992	58
32	0.52992	0.53238	0.53484	0.53730	0.53975	0.54220	0.54464	57
33	0.54464	0.54708	0.54951	0.55194	0.55436	0.55678	0.55919	56
34	0.55919	0.56160	0.56401	0.56641	0.56880	0.57119	0.57358	55
35	0.57358	0.57596	0.57833	0.58070	0.58307	0.58543	0.58779	54
36	0.58779	0.59014	0.59248	0.59482	0.59716	0.59949	0.60182	53
37	0.60182	0.60414	0.60645	0.60876	0.61107	0.61337	0.61566	52
38	0.61566	0.61795	0.62024	0.62251	0.62479	0.62706	0.62932	51
39	0.62932	0.63158	0.63383	0.63608	0.63832	0.64056	0.64279	50
40	0.64279	0.64501	0.64723	0.64945	0.65166	0.65386	0.65606	49
41	0.65606	0.65825	0.66044	0.66262	0.66480	0.66697	0.66913	48
42	0.66913	0.67129	0.67344	0.67559	0.67778	0.67987	0.68200	47
43	0.68200	0.68412	0.68624	0.68835	0.69046	0.69256	0.69466	46
44	0.69466	0.69675	0.69883	0.70091	0.70298	0.70505	0.70711	45
Sines	60′	50′	40′	30′	20′	10′	0′	Degrees
	Cosines							

Degrees	Cosines							Sines
	0′	10′	20′	30′	40′	50′	60′	
0	1.00000	1.00000	0.99998	0.99996	0.99993	0.99989	0.99985	89
1	0.99985	0.99979	0.99973	0.99966	0.99958	0.99949	0.99939	88
2	0.99939	0.99929	0.99917	0.99905	0.99892	0.99878	0.99863	87
3	0.99863	0.99847	0.99831	0.99813	0.99795	0.99776	0.99756	86
4	0.99756	0.99736	0.99714	0.99692	0.99668	0.99644	0.99619	85
5	0.99619	0.99594	0.99567	0.99540	0.99511	0.99482	0.99452	84
6	0.99452	0.99421	0.99390	0.99357	0.99324	0.99290	0.99255	83
7	0.99255	0.99219	0.99182	0.99144	0.99106	0.99067	0.99027	82
8	0.99027	0.98986	0.98944	0.98902	0.98858	0.98814	0.98769	81
9	0.98769	0.98723	0.98676	0.98629	0.98580	0.98531	0.98481	80
10	0.98481	0.98430	0.98378	0.98325	0.98272	0.98218	0.98163	79
11	0.98163	0.98107	0.98050	0.97992	0.97934	0.97875	0.97815	78
12	0.97815	0.97754	0.97692	0.97630	0.97566	0.97502	0.97437	77
13	0.97437	0.97371	0.97304	0.97237	0.97169	0.97100	0.97030	76
14	0.97030	0.96959	0.96887	0.96815	0.96742	0.96667	0.96593	75
15	0.96593	0.96517	0.96440	0.96363	0.96285	0.96206	0.96126	74
16	0.96126	0.96046	0.95964	0.95882	0.95799	0.95715	0.95630	73
17	0.95630	0.95545	0.95459	0.95372	0.95284	0.95195	0.95106	72
18	0.95106	0.95015	0.94924	0.94832	0.94740	0.94646	0.94552	71
19	0.94552	0.94457	0.94361	0.94264	0.94167	0.94068	0.93969	70
20	0.93969	0.93869	0.93769	0.93667	0.93565	0.93462	0.93358	69
21	0.93358	0.93253	0.93148	0.93042	0.92935	0.92827	0.92718	68
22	0.92718	0.92609	0.92499	0.92388	0.92276	0.92164	0.92050	67
23	0.92050	0.91936	0.91822	0.91706	0.91590	0.91472	0.91355	66
24	0.91355	0.91236	0.91116	0.90996	0.90875	0.90753	0.90631	65
25	0.90631	0.90507	0.90383	0.90259	0.90133	0.90007	0.89879	64
26	0.89879	0.89752	0.89623	0.89493	0.89363	0.89232	0.89101	63
27	0.89101	0.88968	0.88835	0.88701	0.88566	0.88431	0.88295	62
28	0.88295	0.88158	0.88020	0.87882	0.87743	0.87603	0.87462	61
29	0.87462	0.87321	0.87178	0.87036	0.86892	0.86748	0.86603	60
30	0.86603	0.86457	0.86310	0.86163	0.86015	0.85866	0.85717	59
31	0.85717	0.85567	0.85416	0.85264	0.85112	0.84959	0.84805	58
32	0.84805	0.84650	0.84495	0.84339	0.84182	0.84025	0.83867	57
33	0.83867	0.83708	0.83549	0.83389	0.83228	0.83066	0.82904	56
34	0.82904	0.82741	0.82577	0.82413	0.82248	0.82082	0.81915	55
35	0.81915	0.81748	0.81580	0.81412	0.81242	0.81072	0.80902	54
36	0.80902	0.80730	0.80558	0.80386	0.80212	0.80038	0.79864	53
37	0.79864	0.79688	0.79512	0.79335	0.79158	0.78980	0.78801	52
38	0.78801	0.78622	0.78442	0.78261	0.78079	0.77897	0.77715	51
39	0.77715	0.77531	0.77347	0.77162	0.76977	0.76791	0.76604	50
40	0.76604	0.76417	0.76229	0.76041	0.75851	0.75661	0.75471	49
41	0.75471	0.75280	0.75088	0.74896	0.74703	0.74509	0.74314	48
42	0.74314	0.74120	0.73924	0.73728	0.73531	0.73333	0.73135	47
43	0.73135	0.72937	0.72737	0.72537	0.72337	0.72136	0.71934	46
44	0.71934	0.71732	0.71529	0.71325	0.71121	0.70916	0.70711	45
Cosines	60′	50′	40′	30′	20′	10′	0′	Degrees
	Sines							

Table 10-4 Natural Trigonometric Functions

Degrees	Tangents							Cotangents
	0′	10′	20′	30′	40′	50′	60′	
0	0.00000	0.00291	0.00582	0.00873	0.01164	0.01455	0.01746	89
1	0.01746	0.02036	0.02328	0.02619	0.02910	0.03201	0.03492	88
2	0.03492	0.03783	0.04075	0.04366	0.04658	0.04949	0.05241	87
3	0.05241	0.05533	0.05824	0.06116	0.06408	0.06700	0.06993	86
4	0.06993	0.07285	0.07578	0.07870	0.08163	0.08456	0.08749	85
5	0.08749	0.09042	0.09335	0.09629	0.09923	0.10216	0.10510	84
6	0.10510	0.10805	0.11099	0.11394	0.11688	0.11983	0.12278	83
7	0.12278	0.12574	0.12869	0.13165	0.13461	0.13758	0.14054	82
8	0.14054	0.14351	0.14648	0.14945	0.15243	0.15540	0.15838	81
9	0.15838	0.16137	0.16435	0.16734	0.17033	0.17333	0.17633	80
10	0.17633	0.17933	0.18233	0.18534	0.18835	0.19136	0.19438	79
11	0.19438	0.19740	0.20042	0.20345	0.20648	0.20952	0.21256	78
12	0.21256	0.21560	0.21864	0.22169	0.22475	0.22781	0.23087	77
13	0.23087	0.23393	0.23700	0.24008	0.24316	0.24624	0.24933	76
14	0.24933	0.25242	0.25552	0.25862	0.26172	0.26483	0.26795	75
15	0.26795	0.27107	0.27419	0.27732	0.28046	0.28360	0.28675	74
16	0.28675	0.28990	0.29305	0.29621	0.29938	0.30255	0.30573	73
17	0.30573	0.30891	0.31210	0.31530	0.31850	0.32171	0.32492	72
18	0.32492	0.32814	0.33136	0.33460	0.33783	0.34108	0.34433	71
19	0.34433	0.34758	0.35085	0.35412	0.35740	0.36068	0.36397	70
20	0.36397	0.36727	0.37057	0.37388	0.37720	0.38053	0.38386	69
21	0.38386	0.38721	0.39055	0.39391	0.39727	0.40065	0.40403	68
22	0.40403	0.40741	0.41081	0.41421	0.41763	0.42105	0.42447	67
23	0.42447	0.42791	0.43136	0.43481	0.43828	0.44175	0.44523	66
24	0.44523	0.44872	0.45222	0.45573	0.45924	0.46277	0.46631	65
25	0.46631	0.46985	0.47341	0.47698	0.48055	0.48414	0.48773	64
26	0.48773	0.49134	0.49495	0.49858	0.50222	0.50587	0.50953	63
27	0.50953	0.51320	0.51688	0.52057	0.52427	0.52798	0.53171	62
28	0.53171	0.53545	0.53920	0.54296	0.54674	0.55051	0.55431	61
29	0.55431	0.55812	0.56194	0.56577	0.56962	0.57348	0.57735	60
30	0.57735	0.58124	0.58513	0.58905	0.59297	0.59691	0.60086	59
31	0.60086	0.60483	0.60881	0.61280	0.61681	0.62083	0.62487	58
32	0.62487	0.62892	0.63299	0.63707	0.64117	0.64528	0.64941	57
33	0.64941	0.65355	0.65771	0.66189	0.66608	0.67028	0.67451	56
34	0.67451	0.67875	0.68301	0.68728	0.69157	0.69588	0.70021	55
35	0.70021	0.70455	0.70891	0.71329	0.71769	0.72211	0.72654	54
36	0.72654	0.73100	0.73547	0.73996	0.74447	0.74900	0.75355	53
37	0.75355	0.75812	0.76272	0.76733	0.77196	0.77661	0.78129	52
38	0.78129	0.78598	0.79070	0.79544	0.80020	0.80498	0.80978	51
39	0.80978	0.81461	0.81946	0.82434	0.82923	0.83415	0.83910	50
40	0.83910	0.84407	0.84906	0.85408	0.85912	0.86419	0.86929	49
41	0.86929	0.87441	0.87955	0.88473	0.88992	0.89515	0.90040	48
42	0.90040	0.90569	0.91099	0.91633	0.92170	0.92709	0.93252	47
43	0.93252	0.93797	0.94345	0.94896	0.95451	0.96008	0.96569	46
44	0.96569	0.97133	0.97700	0.98270	0.98843	0.99420	1.00000	45
Tangents	60′	50′	40′	30′	20′	10′	0′	Degrees
	Cotangents							

Degrees	Cotangents							Tangents
	0′	10′	20′	30′	40′	50′	60′	
0	∞	343.77371	171.88540	114.58865	85.93979	68.75009	57.28996	89
1	57.28996	49.10388	42.96408	38.18846	34.36777	31.24158	28.63625	88
2	28.63625	26.43160	24.54176	22.90377	21.47040	20.20555	19.08114	87
3	19.08114	18.07498	17.16934	16.34986	15.60479	14.92442	14.30067	86
4	14.30067	13.72674	13.19688	12.70621	12.25051	11.82617	11.43005	85
5	11.43005	11.05943	10.71191	10.38540	10.07803	9.78817	9.51436	84
6	9.51436	9.25530	9.00983	8.77689	8.55555	8.34496	8.14435	83
7	8.14435	7.95302	7.77035	7.59575	7.42871	7.26873	7.11537	82
8	7.11537	6.96823	6.82694	6.69116	6.56055	6.43484	6.31375	81
9	6.31375	6.19703	6.08444	5.97576	5.87080	5.76937	5.67128	80
10	5.67128	5.57638	5.48451	5.39552	5.30928	5.22566	5.14455	79
11	5.14455	5.06584	4.98940	4.91516	4.84300	4.77286	4.70463	78
12	4.70463	4.63825	4.57363	4.51071	4.44942	4.38969	4.33148	77
13	4.33148	4.27471	4.21933	4.16530	4.11256	4.06107	4.01078	76
14	4.01078	3.96165	3.91364	3.86671	3.82083	3.77595	3.73205	75
15	3.73205	3.68909	3.64705	3.60588	3.56557	3.52609	3.48741	74
16	3.48741	3.44951	3.41236	3.37594	3.34023	3.30521	3.27085	73
17	3.27085	3.23714	3.20406	3.17159	3.13972	3.10842	3.07768	72
18	3.07768	3.04749	3.01783	2.98869	2.96004	2.93189	2.90421	71
19	2.90421	2.87700	2.85023	2.82391	2.79802	2.77254	2.74748	70
20	2.74748	2.72281	2.69853	2.67462	2.65109	2.62791	2.60509	69
21	2.60509	2.58261	2.56046	2.53865	2.51715	2.49597	2.47509	68
22	2.47509	2.45451	2.43422	2.41421	2.39449	2.37504	2.35585	67
23	2.35585	2.33696	2.31826	2.29984	2.28167	2.26374	2.24604	66
24	2.24604	2.22857	2.21132	2.19430	2.17749	2.16090	2.14451	65
25	2.14451	2.12832	2.11233	2.09654	2.08094	2.06553	2.05030	64
26	2.05030	2.03526	2.02039	2.00569	1.99116	1.97680	1.96261	63
27	1.96261	1.94858	1.93470	1.92089	1.90741	1.89400	1.88073	62
28	1.88073	1.86760	1.85462	1.84177	1.82907	1.81649	1.80405	61
29	1.80405	1.79174	1.77955	1.76749	1.75556	1.74375	1.73205	60
30	1.73205	1.72047	1.70901	1.69766	1.68643	1.67530	1.66428	59
31	1.66428	1.65337	1.64256	1.63185	1.62125	1.61074	1.60033	58
32	1.60033	1.59002	1.57981	1.56969	1.55966	1.54972	1.53987	57
33	1.53987	1.53010	1.52043	1.51084	1.50133	1.49190	1.48256	56
34	1.48256	1.47330	1.46411	1.45501	1.44598	1.43703	1.42815	55
35	1.42815	1.41934	1.41061	1.40195	1.39336	1.38484	1.37638	54
36	1.37638	1.36800	1.35968	1.35142	1.34323	1.33511	1.32704	53
37	1.32704	1.31904	1.31110	1.30323	1.29541	1.28764	1.27994	52
38	1.27994	1.27230	1.26471	1.25717	1.24969	1.24227	1.23490	51
39	1.23490	1.22758	1.22031	1.21310	1.20593	1.19882	1.19175	50
40	1.19175	1.18474	1.17777	1.17085	1.16398	1.15715	1.15037	49
41	1.15037	1.14363	1.13694	1.13029	1.12369	1.11713	1.11061	48
42	1.11061	1.10414	1.09770	1.09131	1.08496	1.07864	1.07237	47
43	1.07237	1.06613	1.05994	1.05378	1.04766	1.04158	1.03553	46
44	1.03553	1.02952	1.02355	1.01761	1.01170	1.00583	1.00000	45
Cotangents	60′	50′	40′	30′	20′	10′	0′	Degrees
	Tangents							

Table 10-4 Natural Trigonometric Functions