Superconducting State generated by Cooper Pairs bound by Intensified Gravitational Interaction

Fran De Aquino
Maranhao State University, Physics Department, S.Luis/MA, Brazil.

We show that by *intensifying the gravitational interaction* between electron pairs it is possible to produce pair binding energies on the order of 10^{-1}eV, enough to keep electron's pairs (*Cooper Pairs*) at *ambient temperatures*. By means of this method, metals can be transformed into superconductors at ambient temperature.

Key words: Modified theories of gravity, Theories and models of superconducting state, superconducting materials, Nonconventional mechanisms.
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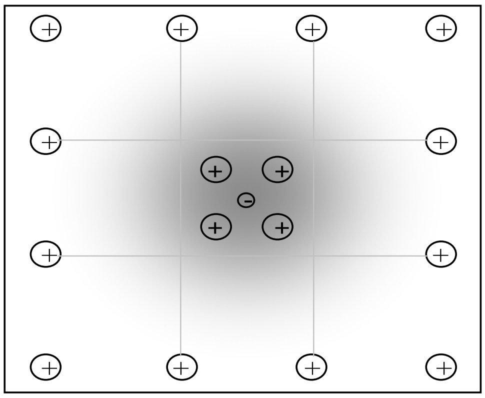
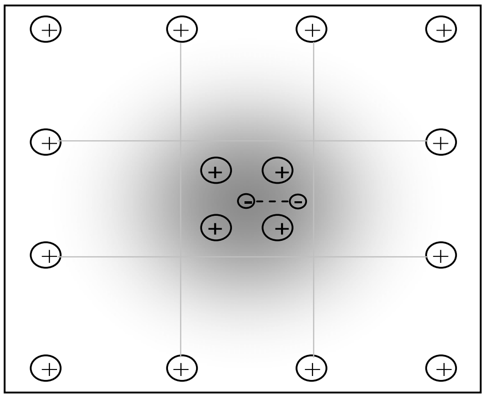
1. Introduction

A pair of weakly bound electrons in a superconductor is called *Cooper pair*; it was first described in 1956 by Leon Cooper [1]. As showed by Cooper, an attraction between electrons in a metal can cause a paired state of electrons to have a lower energy than the Fermi energy, which implies that the pair is *bound*. In conventional superconductors, this attraction is due to the electron–phonon interaction. *The Cooper pair state is responsible for superconductivity, as described in* the BCS theory developed by John Bardeen, John Schrieffer and Leon Cooper for which they shared the 1972 Nobel Prize [2].

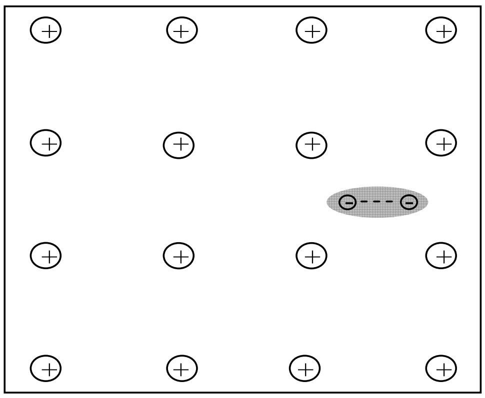
In spite of Cooper pairing to be a quantum effect the reason for the pairing can be seen from a simplified classical explanation [3]. In order to understand how an attraction between two electrons can occur, it is necessary to consider the interaction with the positive ions lattice of the metal. Usually an electron in a metal behaves as a free particle. Its negative charge causes attraction between *the positive ions that make up the rigid lattice of the metal*. This attraction distorts the ion lattice, moving the ions slightly toward the electron, *increasing the positive charge density of the lattice in the local* (See gray glow in Fig.1 (a)). Then, another electron is attracted to the positive charge density (gray glow) created by the first electron distorting the lattice around itself. This attraction can overcome the electrons' repulsion due to their negative charge and create a binding between the two



(a)



(b)



(c)
Fig. 1 – Cooper Pair Formation

electrons (See Fig.1 (b)). *The electrons can then travel through the lattice* as *a single entity*, known as a *Cooper Pair* (See Fig.1 (c)). While conventional conduction is resisted by thermal vibrations within the lattice, Cooper Pairs carry the supercurrent relatively unresisted by thermal vibrations.

The energy of the pairing interaction is quite weak, of the order of 10^{-3}eV, and thermal energy can easily break the pairs. *So only at low temperatures, are a significant number of the electrons in a metal in Cooper pairs*.

Here is showed that, by *intensifying the gravitational interaction* * [4] between electrons pairs, it is possible to produce pair binding energies on the order of 10^{-1}eV, enough to keep them paired at *ambient temperatures*. Thus, by this way, metals at ambient temperature can have a significant number of the electrons in Cooper pairs, transforming such metals in superconductors at ambient temperature.

2. Theory

The quantization of gravity showed that the *gravitational mass* m_g and the *inertial mass* m_i are correlated by means of the following factor [4]:

$$\chi = \frac{m_g}{m_{i0}} = \left\{1 - 2\left[\sqrt{1 + \left(\frac{\Delta p}{m_{i0}c}\right)^2} - 1\right]\right\} \qquad (1)$$

where m_{i0} is the *rest* inertial mass of the particle and Δp is the variation in the particle's *kinetic momentum*; c is the speed of light.

When Δp is produced by the absorption of a photon with wavelength λ, it is expressed by $\Delta p = h/\lambda$. In this case, Eq. (1) becomes

$$\frac{m_g}{m_{i0}} = \left\{1 - 2\left[\sqrt{1 + \left(\frac{h/m_{i0}c}{\lambda}\right)^2} - 1\right]\right\}$$
$$= \left\{1 - 2\left[\sqrt{1 + \left(\frac{\lambda_0}{\lambda}\right)^2} - 1\right]\right\} \qquad (2)$$

where $\lambda_0 = h/m_{i0}c$ is the *DeBroglie wavelength* for the particle with *rest* inertial mass m_{i0}.

In general, the *momentum* variation Δp is expressed by $\Delta p = F\Delta t$ where F is the applied force during a time interval Δt. Note that there is no restriction concerning the *nature* of the force, i.e., it can be mechanical, electromagnetic, etc. For example, we can look on the *momentum* variation Δp as due to absorption or emission of *electromagnetic energy* by the particle.

This means that, by means of electromagnetic fields, the *gravitational mass* can be decreased down to become negative and *increased* (*independently* of the inertial mass m_i). In this way, *the gravitational forces can be intensified*. Consequently, we can use, for example, oscillating magnetic fields in order to *intensify the gravitational interaction* between electrons pairs, in order to produce pair binding energies enough to keep them paired at *ambient temperatures*. We will show that the magnetic field used in this case must have extremely-low frequency (ELF).

From Electrodynamics we know that when an electromagnetic wave with frequency f and velocity c incides on a material with relative permittivity ε_r, relative magnetic permeability μ_r and electrical conductivity σ, its *velocity is reduced* to $v = c/n_r$ where n_r is the index of refraction of the material, given by [5]

$$n_r = \frac{c}{v} = \sqrt{\frac{\varepsilon_r \mu_r}{2}\left(\sqrt{1 + (\sigma/\omega\varepsilon)^2} + 1\right)} \qquad (3)$$

* De Aquino, F. (2008) *Process and Device for* Controlling the Locally the Gravitational Mass and the Gravity Acceleration, BR Patent Number: PI0805046-5, July 31, 2008.

If $\sigma >> \omega\varepsilon$, $\omega = 2\pi f$, Eq. (3) reduces to

$$n_r = \sqrt{\frac{\mu_r \sigma}{4\pi\varepsilon_0 f}} \qquad (4)$$

Thus, the wavelength of the incident radiation (See Fig. 2) becomes

$$\lambda_{\text{mod}} = \frac{v}{f} = \frac{c/f}{n_r} = \frac{\lambda}{n_r} = \sqrt{\frac{4\pi}{\mu f \sigma}} \qquad (5)$$

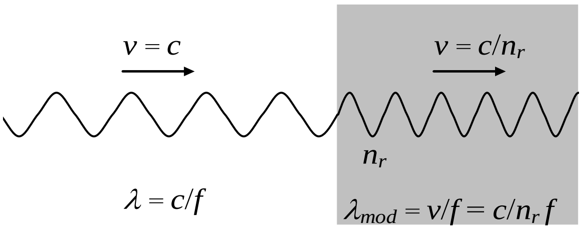


Fig. 2 – *Modified Electromagnetic Wave*. The wavelength of the electromagnetic wave can be strongly reduced, but its frequency remains the same.

If a lamina with thickness equal to ξ contains n atoms/m^3, then the number of atoms per area unit is $n\xi$. Thus, if the electromagnetic radiation with frequency f incides on an area S of the lamina it reaches $nS\xi$ atoms. If it incides on the *total area of the lamina*, S_f, then the total number of atoms reached by the radiation is $N = nS_f\xi$. The number of atoms per unit of volume, n, is given by

$$n = \frac{N_0 \rho}{A} \qquad (6)$$

where $N_0 = 6.02 \times 10^{26}\, atoms/kmole$ is the Avogadro's number; ρ is the matter density of the lamina (in *kg/m*^{3}) and A is the molar mass(*kg/kmole*).

When an electromagnetic wave incides on the lamina, it strikes N_f front atoms, where $N_f \cong (nS_f)\phi_m$, ϕ_m is the "diameter" of the atom. Thus, the electromagnetic wave incides effectively on an area $S = N_f S_m$, where $S_m = \frac{1}{4}\pi\phi_m^2$ is the cross section area of one atom. After these collisions, it carries out $n_{collisions}$ with the other atoms (See Fig.3).

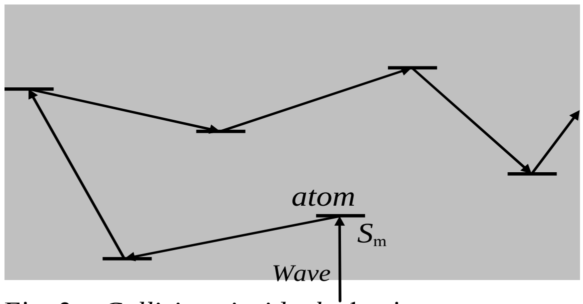


Fig. 3 – *Collisions inside the* lamina.

Thus, the total number of collisions in the volume $S\xi$ is

$$N_{collisions} = N_f + n_{collisions} = n_l S\phi_m + (n_l S\xi - n_m S\phi_m) = \\ = n_m S\xi \qquad (7)$$

The power density, D, of the radiation on the lamina can be expressed by

$$D = \frac{P}{S} = \frac{P}{N_f S_m} \qquad (8)$$

We can express the *total mean number of collisions in each atom*, n_1, by means of the following equation

$$n_1 = \frac{n_{total\ photons} N_{collisions}}{N} \qquad (9)$$

Since in each collision a *momentum* h/λ is transferred to the atom, then the *total momentum* transferred to the lamina will be $\Delta p = (n_1 N)h/\lambda$. Therefore, in accordance with Eq. (1), we can write that

$$\frac{m_{g(l)}}{m_{i0(l)}} = \left\{1 - 2\left[\sqrt{1 + \left[(n_1 N)\frac{\lambda_0}{\lambda}\right]^2} - 1\right]\right\} =$$

$$= \left\{1 - 2\left[\sqrt{1 + \left[n_{total\ photons} N_{collisions}\frac{\lambda_0}{\lambda}\right]^2} - 1\right]\right\} \qquad (10)$$

Since Eq. (7) gives $N_{collisions} = n_l S\xi$, we get

$$n_{total\ \ photons}N_{collisions} = \left(\frac{P}{hf^2}\right)\left(n_l S\xi\right) \qquad (11)$$

Substitution of Eq. (11) into Eq. (10) yields

$$\frac{m_{g(l)}}{m_{i0(l)}} = \left\{1 - 2\left[\sqrt{1 + \left[\left(\frac{P}{hf^2}\right)\left(n_l S\xi\right)\frac{\lambda_0}{\lambda}\right]^2} - 1\right]\right\} \qquad (12)$$

Substitution of P given by Eq. (8) into Eq. (12) gives

$$\frac{m_{g(l)}}{m_{i0(l)}} = \left\{1 - 2\left[\sqrt{1 + \left[\left(\frac{N_f S_m D}{f^2}\right)\left(\frac{n_l S\xi}{m_{i0(l)}c}\right)\frac{1}{\lambda}\right]^2} - 1\right]\right\} \qquad (13)$$

Substitution of $N_f \cong \left(n_l S_f\right)\phi_m$ and $S = N_f S_m$ into Eq. (13) results

$$\frac{m_{g(l)}}{m_{i0(l)}} = \left\{1 - 2\left[\sqrt{1 + \left[\left(\frac{n_l^3 S_f^2 S_m^2 \phi_m^2 \xi D}{m_{i0(l)}cf^2}\right)\frac{1}{\lambda}\right]^2} - 1\right]\right\} \qquad (14)$$

where $m_{i0(l)} = \rho_{(l)}V_{(l)}$.

Now, considering that the lamina is inside an ELF electromagnetic field with E and B, then we can write that [6]

$$D = \frac{n_{r(l)}E^2}{2\mu_0 c} \qquad (15)$$

Substitution of Eq. (15) into Eq. (14) gives

$$\frac{m_{g(l)}}{m_{i0(l)}} = \left\{1 - 2\left[\sqrt{1 + \left[\left(\frac{n_{r(l)}n_l^3 S_f^2 S_m^2 \phi_m^2 \xi E^2}{2\mu_0 m_{i0(l)}c^2 f^2}\right)\frac{1}{\lambda}\right]^2} - 1\right]\right\} \qquad (16)$$

Note that $E = E_m \sin \omega t$. The average value for E^2 is equal to $\frac{1}{2}E_m^2$ because E varies sinusoidaly (E_m is the maximum value for E). On the other hand, $E_{rms} = E_m/\sqrt{2}$. Consequently we can replace E^4 for E_{rms}^4.

Thus, for $\lambda = \lambda_{\text{mod}}$, the equation above can be rewritten as follows

$$\frac{m_{g(l)}}{m_{i0(l)}} = \left\{1 - 2\left[\sqrt{1 + \left[\left(\frac{n_{r(l)}n_l^3 S_f^2 S_m^2 \phi_m^2 \xi E_{rms}^2}{2\mu_0 m_{i0(l)}c^2 f^2}\right)\frac{1}{\lambda_{\text{mod}}}\right]^2} - 1\right]\right\} \qquad (17)$$

Electrodynamics tells us that $E_{rms} = vB_{rms} = \left(c/n_{r(l)}\right)B_{rms}$. Substitution of this expression into Eq. (17) gives

$$\chi = \frac{m_{g(l)}}{m_{i0(l)}} = \left\{1 - 2\left[\sqrt{1 + \frac{n_l^6 S_f^4 S_m^4 \phi_m^4 \xi^2 B_{rms}^4}{4\mu_0^2 m_{i0(l)}^2 f^4 \lambda_{\text{mod}}^2 n_{r(l)}^2}} - 1\right]\right\} \qquad (18)$$

Since $\lambda_{\text{mod}} = \lambda/n_{r(l)}$ then Eq. (18) can be rewritten in the following form

$$\chi = \frac{m_{g(l)}}{m_{i0(l)}} = \left\{1 - 2\left[\sqrt{1 + \frac{n_l^6 S_f^4 S_m^4 \phi_m^4 \xi^2 B_{rms}^4}{4\mu_0^2 m_{i0(l)}^2 c^2 f^2}} - 1\right]\right\} \qquad (19)$$

In order to calculate the expressions of χ_{Be} for the particular case of a *free electron* inside a conductor, subjected to an external magnetic field B_{rms} with frequency f, we must consider the interaction with the *positive ions that make up the rigid lattice of the metal*.

The negative charge of the free electron causes attraction between *the positive ions lattice of the metal*. This attraction distorts the ion lattice, moving the ions slightly toward the electron, *increasing the positive charge density of the lattice in the local* (See gray glow in Fig.1 (a)). Then, another electron is attracted to the positive charge density (gray glow) created by the first electron distorting the lattice around itself, *which produces a strong attraction upon the electron* deforming its surface as showed in Fig. 4. Under these circumstances, *the volume of the electron does not vary*, but its external surface is strongly increased, becomes equivalent to the external area of a sphere with radius $r_{xe} >> r_e$ (r_e is the radius of the *free* electron out of the ions “gage” showed in Fig. 1 (a)). Based on such conclusions, we substitute in Eq.(19) n_l by

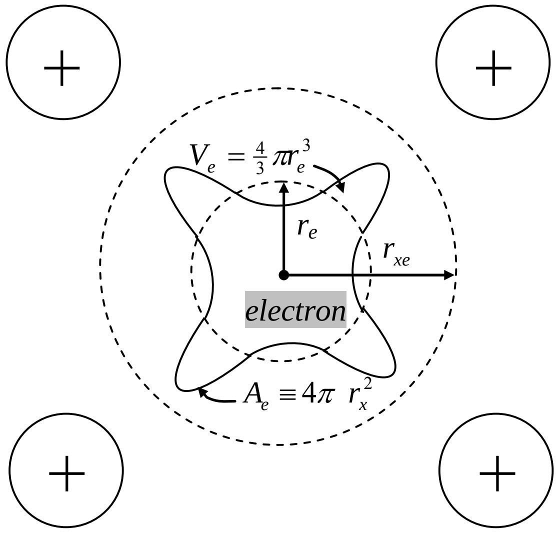


Fig. 4 – *Schematic diagram of Electrons' structure inside the ion lattice.* The positive ions lattice around the electron produces a strong attraction upon the electron deforming its surface. The volume of the electron does not vary, but its external surface is increased and becomes equivalent to the area of a sphere with radius $r_{xe} >> r_e$.

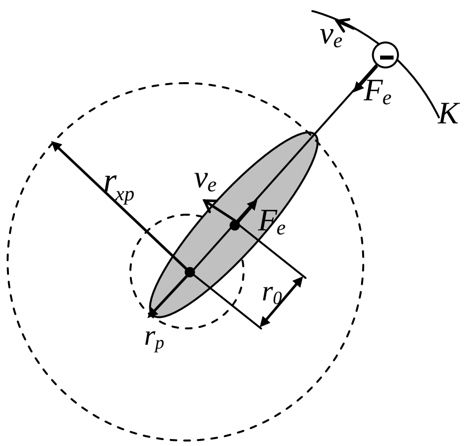


Fig. 5 – *The deformation of the proton.*

$1/V_e = 1/\frac{4}{3}\pi r_e^3$, S_f by $(SSA_e)\rho_e V_e$ (SSA_e is the *specific surface area* for electrons in this case: $SSA_e = \frac{1}{2}A_e/m_e = \frac{1}{2}A_e/\rho_e V_e = 2\pi r_{xe}^2/\rho_e V_e$), S_m by $S_e = \pi r_{xe}^2$, ξ by $\phi_m = 2r_{xe}$ and $m_{i0(l)}$ by m_e. The result is

$$\chi_{Be} = \left\{1 - 2\left[\sqrt{1 + \frac{45.56\pi^2 r_{xe}^{22} B_{rms}^4}{c^2 \mu_0^2 m_e^2 r_e^{18} f^2}} - 1\right]\right\} \qquad (20)$$

In order to calculate the value of r_{xe} we start considering a hydrogen atom, where the electron spins around the proton with a velocity $v_e = 3\times 10^6 m.s^{-1}$. The electrical force acting on the proton is $F_e = e^2/4\pi\varepsilon_0\, r_1^2$, which is equal to the centrifuge force $F_c = m_p \omega_e^2 r_0$ where ω_e is the angular velocity of the electron and r_0 is the distance between the inertial center of the proton and the center of the moving proton (See Fig. 5, where we conclude that $2(r_0 + r_p) = r_{xp} + r_p$; r_{xp} is the radius of the sphere whose external area is equivalent to the increased area of the proton). Thus, we get $r_0 = \frac{1}{2}(r_{xp} - r_p)$. Substitution of this value into expression of $F_c = F_e$ gives

$$r_{xp} = \frac{e^2}{4\pi\varepsilon_0 m_p v_e^2} + r_p = 3.2\times 10^{-14} m$$

Therefore, we can write that $r_{xp} = k_{xp} r_p$, where

$$k_{xp} = \frac{r_{xp}}{r_p} = 25.6$$

The *electron is similarly deformed by the relative movement of the proton in respect to the electron*. In this case, by analogy, we can write that

$$r_{xe} = \frac{e^2}{4\pi\varepsilon_0 m_e v_e^2} + r_e = 6.4\times 10^{-11} m$$

and $r_{xe} = k_{xe} r_e$, where r_{xe} is the radius of the sphere whose external area is equivalent to the increased area of the electron. The radius of *free* electron is $r_e = 6.87\times 10^{-14} m$ (See *Appendix* A). However, for electrons in the atomic eletrosphere the value of r_e can be calculated starting from Quantum Mechanics.

The wave packet that describes the electron satisfies an *uncertainty principle* $\left(\Delta p \Delta x \geq \frac{1}{2}\hbar\right)$, where $\Delta p = \hbar \Delta k$ and Δk is the approximate extension of the wave packet. Thus, we can write that $\left(\Delta k \Delta x \geq \frac{1}{2}\right)$. For the ``square" packet the full width in k is $\Delta k = 2\pi/\lambda_0$ ($\lambda_0 = h/m_e c$ is the average wavelength). The width in x is a little harder to define, but, lets use the first node in the probability found at $(2\pi/\lambda_0)x/2 = \pi$ or $x = \lambda_0$. So, the width of the wave packet is twice this or $\Delta x = 2\lambda_0$. Obviously, $2r_e$ cannot be greater than Δx, i.e., r_e *must be smaller and close to* $\lambda_0 = h/m_e c = 2.43\times10^{-12}m$. Then, assuming that $r_e \cong 2.4\times10^{-12}m$, we get

$$k_{xe} = \frac{r_{xe}}{r_e} = 26.6$$

Note that $k_{xe} \cong k_{xp}$. In the case *of electrons inside the ion lattice* (See Fig. 4), we can note that, in spite of the electron speed v_e be null, the deformations are similar, in such way that, in this case, we can take the values above.

Substitution of these values into Eq. (20) gives

$$\chi_{Be} = \left\{1 - 2\left[\sqrt{1 + 3.8\times10^{57}\frac{k_{xe}^{22} r_e^4 B_{rms}^4}{f^2}} - 1\right]\right\} = \left\{1 - 2\left[\sqrt{1 + 2.8\times10^{42}\frac{B_{rms}^4}{f^2}} - 1\right]\right\} \quad (21)$$

Similarly, in the case of *proton* and *neutron* we can write that

$$\chi_{Bp} = \left\{1 - 2\left[\sqrt{1 + \frac{45.56\pi^2 k_{xp}^{22} r_p^4 B_{rms}^4}{c^2 \mu_0^2 m_p^2 f^2}} - 1\right]\right\} \quad (22)$$

$$\chi_{Bn} = \left\{1 - 2\left[\sqrt{1 + \frac{45.56\pi^2 r_n^4 B_{rms}^4}{\mu_0^2 m_n^2 c^2 f^2}} - 1\right]\right\} \quad (23)$$

In the case of the neutron, $k_{xn} = 1$ due to its electric charge be null. The radius of *protons inside the atoms* (nuclei) is $r_p = 1.2\times10^{-15}m$ [7,8], $r_n \cong r_p$, then we obtain from Eqs. (22) and (23) following expressions:

$$\chi_{Bp} = \left\{1 - 2\left[\sqrt{1 + 2.2\times10^{22}\frac{B_{rms}^4}{f^2}} - 1\right]\right\} \quad (24)$$

$$\chi_{Bn} = \left\{1 - 2\left[\sqrt{1 + 2.35\times10^{9}\frac{B_{rms}^4}{f^2}} - 1\right]\right\} \quad (24a)$$

Since $m_{ge} = \chi_{Be} m_e$, $m_{gp} = \chi_{Bp} m_p$ and $m_{gn} = \chi_{Bn} m_n$, it easy to see, by means of Eqs. (21), (24) and (24a), that m_{ge} is much greater than m_{gp} and m_{gp}. This means that, in the calculation of the gravitational force F_g (between the positive ions + electron and the external electron), we can disregard the effects of the gravitational masses of the ions. Thus, the expression of F_g reduces to the expression of the gravitational forces between the two electrons, i.e.,

$$F_g = -G\frac{m_{ge}^2}{r^2} = -\chi_{Be}^2 G\frac{m_e^2}{r^2} \quad (25)$$

For the creation of the Cooper Pairs F_g must overcome the electrons' repulsion due to their negative charge $\left(e^2/4\pi\varepsilon_0 r^2\right)$. Thus, we must have $\chi_{Be}^2 G m_e^2 > e^2/4\pi\varepsilon_0$ or

$$\chi_{Be} > \frac{(e/m_e)}{\sqrt{4\pi\varepsilon_0 G}} = -2\times10^{21} \quad (26)$$

For the Cooper Pairs *not be destructed by the thermal vibrations* due to the temperature T, we must have $\chi_{Be}^2 G m_e^2/r > kT$ whence we conclude that $T < \chi_{Be}^2 G m_e^2/r$. Consequently, the *transition temperature*, T_c, can be expressed by the following expression

$$T_c = \frac{\chi_{Be}^2 G m_e^2}{k\xi} \qquad (27)$$

where ξ is the size of the Cooper pair, which is given by the *coherence length* of the Cooper-pair wavefunction. It is known that the coherence length is typically 1000 Å (though it can be as small as 30Å in the copper oxides). The coherence length of the Cooper-pair in Aluminum superconductor is quite large ($\xi \cong 1\ micron$ [9]). Substitution of this value into Eq. (27) gives

$$T_c = 4\times10^{-42}\chi_{Be}^2 \qquad (28)$$

For $T_c = 400K\ (\sim 127°C)$ we obtain

$$\chi_{Be} = -1\times10^{22} \qquad (29)$$

By comparing (29) with (26), we can conclude that this value of χ_e is sufficient *for the creation* of the Cooper Pairs, and also in order that they do not *be destructed by the thermal vibrations* due to the temperature up to $T_c = 400K\ (\sim 127°C)$.

In order to calculate the intensity of the magnetic field B_m with frequency f, necessary to produce the value given by Eq.(29), it is necessary the substitution of Eq. (29) into Eq. (21). Thus, we get

$$\left\{1-2\left[\sqrt{1+2.8\times10^{42}\frac{B_{rms}^4}{f^2}}-1\right]\right\}\cong -1\times10^{22} \qquad (30)$$

For $f = 2Hz$ the value of B_{rms} is

$$B_{rms} > 3T$$

Therefore, if a magnetic field with frequency $f = 2Hz$ and intensity $B_{rms} > 3T$ † is applied upon an Aluminum wire it becomes superconductor at ambient temperature ($T_c = 400K\ (\sim 127°C)$). Note that the magnetic field is used only during a time interval sufficient to transform the Aluminum into a superconductor. This means that the process is a some sort of "magnetization" that transforms a conductor into a "permanent" superconductor. After the "magnetization" the magnetic field can be turned off, similarly to the case of "magnetization" that transforms an iron rod into a "permanent" magnet.

† Modern magnetic resonance imaging systems work with magnetic fields up to $8T$ [10, 11].

Appendix A: *The "Geometrical Radii" of Electron and Proton*

It is known that the frequency of oscillation of a simple spring oscillator is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \qquad (A1)$$

where m is the inertial mass attached to the spring and K is the spring constant (in N·m^{-1}). In this case, the restoring *force* exerted by the spring *is linear* and given by

$$F = -Kx \qquad (A2)$$

where x is the displacement from the equilibrium position.

Now, consider the gravitational force: For example, above the surface of the Earth, the force follows the familiar Newtonian function, i.e., $F = -GM_{g\oplus}m_g/r^2$, where $M_{g\oplus}$ is the mass of Earth, m_g is the gravitational mass of a particle and r is the distance between the centers. *Below* Earth's surface the *force is linear* and given by

$$F = -\frac{GM_{g\oplus}m_g}{R_\oplus^3}r \qquad (A3)$$

where $R_\oplus$ is the radius of Earth.

By comparing (A3) with (A2) we obtain

$$\frac{K}{m_g} = \frac{K}{\chi\, m} = \frac{GM_{g\oplus}}{R_\oplus^3}\left(\frac{r}{x}\right) \qquad (A4)$$

Making $x = r = R_\oplus$, and substituting (A4) into (A1) gives

$$f = \frac{1}{2\pi}\sqrt{\frac{GM_{g\oplus}\chi}{R_\oplus^3}} \qquad (A5)$$

In the case of an *electron* and a *positron*, we substitute $M_{g\oplus}$ by m_{ge}, χ by χ_e and $R_\oplus$ by R_e, where R_e is the radius of electron (or positron). Thus, Eq. (A5) becomes

$$f = \frac{1}{2\pi}\sqrt{\frac{Gm_{ge}\chi_e}{R_e^3}} \qquad (A6)$$

The value of χ_e varies with the density of energy [4]. When the electron and the positron are distant from each other and the local density of energy is small, the value of χ_e becomes very close to 1. However, *when the electron and the positron are penetrating one another*, the energy densities in each particle become very strong due to the proximity of their electrical charges e and, consequently, the value of χ_e strongly increases. In order to calculate the value of χ_e under these conditions ($x = r = R_e$), we start from the expression of correlation between *electric charge* q and *gravitational mass*, obtained in a previous work [4]:

$$q = \sqrt{4\pi\varepsilon_0 G}\;\, m_{g(imaginary)}\, i \qquad (A7)$$

where $m_{g(imaginary)}$ is the *imaginary* gravitational mass, and $i = \sqrt{-1}$.

In the case of *electron*, Eq. (A7) gives

$$\begin{aligned} q_e &= \sqrt{4\pi\varepsilon_0 G}\;\, m_{ge(imaginary)}\, i = \\ &= \sqrt{4\pi\varepsilon_0 G}\left(\chi_e m_{i0e(imaginary)} i\right) = \\ &= \sqrt{4\pi\varepsilon_0 G}\left(-\chi_e \tfrac{2}{\sqrt{3}} m_{i0e(real)} i^2\right) = \\ &= \sqrt{4\pi\varepsilon_0 G}\left(\tfrac{2}{\sqrt{3}}\chi_e m_{i0e(real)}\right) = -1.6\times10^{-19}C \qquad (A8) \end{aligned}$$

where we obtain

$$\chi_e = -1.8\times10^{21} \qquad (A9)$$

This is therefore, the value of χ_e increased by the strong density of energy produced by the electrical charges e of the two particles, under previously mentioned conditions.

Given that $m_{ge} = \chi_e m_{i0e}$, Eq. (A6) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{G\chi_e^2 m_{i0e}}{R_e^3}} \qquad (A10)$$

From Quantum Mechanics, we know that

$$hf = m_{i0}c^2 \qquad (A11)$$

where h is the Planck's constant. Thus, in the case of $m_{i0} = m_{i0e}$ we get

$$f = \frac{m_{i0e}c^2}{h} \qquad (A12)$$

By comparing (A10) and (A12) we conclude that

$$\frac{m_{i0e}c^2}{h} = \frac{1}{2\pi}\sqrt{\frac{G\chi_e^2 m_{i0e}}{R_e^3}} \qquad (A13)$$

Isolating the radius R_e, we get:

$$R_e = \left(\frac{G}{m_{i0e}}\right)^{\frac{1}{3}}\left(\frac{\chi_e h}{2\pi\ c^2}\right)^{\frac{2}{3}} = 6.87\times10^{-14}m \quad (A14)$$

Compare this value with the *Compton sized electron*, which predicts $R_e = 3.86\times10^{-13}m$ and also with standardized result recently obtained of $R_e = 4-7\times10^{-13}m$ [12].

In the case of *proton*, we have

$$q_p = \sqrt{4\pi\varepsilon_0 G}\ m_{gp(imaginary)}\, i =$$
$$= \sqrt{4\pi\varepsilon_0 G}\left(\chi_p m_{i0p(imaginary)} i\right) =$$
$$= \sqrt{4\pi\varepsilon_0 G}\left(-\chi_p \tfrac{2}{\sqrt{3}} m_{i0p(real)} i^2\right) =$$
$$= \sqrt{4\pi\varepsilon_0 G}\left(\tfrac{2}{\sqrt{3}} \chi_p m_{i0p(real)}\right) = -1.6\times10^{-19}C \quad (A15)$$

where we obtain

$$\chi_p = -9.7\times10^{17} \qquad (A16)$$

Thus, the result is

$$R_p = \left(\frac{G}{m_{i0p}}\right)^{\frac{1}{3}}\left(\frac{\chi_p h}{2\pi\ c^2}\right)^{\frac{2}{3}} = 3.72\times10^{-17}m \quad (A17)$$

Note that these radii, given by Equations $(A14)$ and $(A17)$, are the radii of *free* electrons and *free* protons (when the particle and antiparticle (in isolation) penetrate themselves mutually).

Inside the atoms (nuclei) the radius of protons is well-known. For example, protons, as the hydrogen nuclei, have a radius given by $R_p \cong 1.2\times10^{-15}m$ [7, 8]. The strong increase in respect to the value given by Eq. (A17) is due to the interaction with the electron of the atom.

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