

Artificial Gravitational Lenses

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We show that it is possible to produce gravitational lenses at laboratory scale by means of a toroidal device which strongly intensifies the radial gravitational acceleration at its nucleus, and can make the acceleration repulsive besides attractive. This means that a light flux through the toroid can become convergent or divergent from its central axis. These lenses are similar to optical lenses and can be very useful for telescopes, microscopes, and for the concentration of solar light in order to convert solar energy into thermal energy.

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1. Introduction

It is known that Gravitational fields can bend light. This effect was confirmed in 1919 during a solar eclipse, when Arthur Eddington observed the light from stars passing close to the sun was slightly bent, so that stars appeared slightly out of position [1]. Einstein realized that a massive astronomical object can bend light making what is called a *gravitational lens*. The *gravitational lensing* is one of the predictions of Einstein's general theory of relativity. Although this phenomenon was first mentioned in 1924 by Orest Chwolson [2], the effect is more commonly associated with Einstein, who published a more famous article on the subject in 1936 [3, 4].

Here we show that it is possible to produce gravitational lenses at laboratory scale, by means of a toroidal device which strongly intensifies the radial *gravitational acceleration* at its nucleus, and can make the acceleration repulsive besides attractive* [5]. This means that a light flux through the toroid can become convergent or divergent from its central axis. These lenses are similar to optical lenses and can be very useful for telescopes, microscopes, and for the concentration of solar light in order to convert solar energy into thermal energy.

2. Theory

From the quantization of gravity it follows that the *gravitational mass* m_g and the *inertial mass* m_i are correlated by means of the following factor [5]:

$$\chi = \frac{m_g}{m_{i0}} = \left\{ 1 - 2 \left[\sqrt{1 + \left(\frac{\Delta p}{m_{i0} c} \right)^2} - 1 \right] \right\} \quad (1)$$

where m_{i0} is the *rest inertial mass* of the particle and Δp is the variation in the particle's *kinetic momentum*; c is the speed of light.

When Δp is produced by the absorption of a photon with wavelength λ , it is expressed by $\Delta p = h/\lambda$. In this case, Eq. (1) becomes

$$\begin{aligned} \frac{m_g}{m_{i0}} &= \left\{ 1 - 2 \left[\sqrt{1 + \left(\frac{h/m_{i0}c}{\lambda} \right)^2} - 1 \right] \right\} \\ &= \left\{ 1 - 2 \left[\sqrt{1 + \left(\frac{\lambda_0}{\lambda} \right)^2} - 1 \right] \right\} \end{aligned} \quad (2)$$

where $\lambda_0 = h/m_{i0}c$ is the *De Broglie wavelength* for the particle with *rest inertial mass* m_{i0} .

It has been shown that there is an additional effect - *Gravitational Shielding* effect - produced by a substance whose gravitational mass was reduced or made negative [6]. The effect extends beyond

* De Aquino, F. (2008) *Process and Device for Controlling the Locally the Gravitational Mass and the Gravity Acceleration*, BR Patent Number: PI0805046-5, July 31, 2008.

substance (gravitational shielding), up to a certain distance from it (along the central axis of gravitational shielding). This effect shows that in this region the gravity acceleration, g_1 , is reduced at the same proportion, i.e., $g_1 = \chi_1 g$ where $\chi_1 = m_g/m_{i0}$ and g is the gravity acceleration *before* the gravitational shielding). Consequently, *after a second gravitational shielding*, the gravity will be given by $g_2 = \chi_2 g_1 = \chi_1 \chi_2 g$, where χ_2 is the value of the ratio m_g/m_{i0} for the *second* gravitational shielding. In a generalized way, we can write that after the n th gravitational shielding the gravity, g_n , will be given by

$$g_n = \chi_1 \chi_2 \chi_3 \dots \chi_n g \quad (3)$$

This possibility shows that, by means of a battery of gravitational shieldings, we can strongly intensify the gravitational acceleration.

From Electrodynamics we know that when an electromagnetic wave with frequency f and velocity c incides on a material with relative permittivity ϵ_r , relative magnetic permeability μ_r and electrical conductivity σ , its *velocity is reduced* to $v = c/n_r$, where n_r is the index of refraction of the material, given by [7]

$$n_r = \frac{c}{v} = \sqrt{\frac{\epsilon_r \mu_r}{2} \left(\sqrt{1 + (\sigma/\omega\epsilon)^2} + 1 \right)} \quad (4)$$

If $\sigma \gg \omega\epsilon$, $\omega = 2\pi f$, Eq. (4) reduces to

$$n_r = \sqrt{\frac{\mu_r \sigma}{4\pi\epsilon_0 f}} \quad (5)$$

Thus, the wavelength of the incident radiation (See Fig. 1) becomes

$$\lambda_{\text{mod}} = \frac{v}{f} = \frac{c/f}{n_r} = \frac{\lambda}{n_r} = \sqrt{\frac{4\pi}{\mu f \sigma}} \quad (6)$$

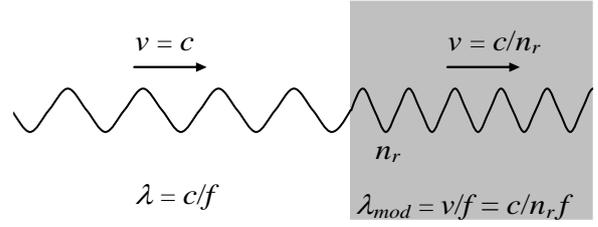


Fig. 1 – *Modified Electromagnetic Wave*. The wavelength of the electromagnetic wave can be strongly reduced, but its frequency remains the same.

If a lamina with thickness equal to ξ contains n atoms/ m^3 , then the number of atoms per area unit is $n\xi$. Thus, if the electromagnetic radiation with frequency f incides on an area S of the lamina it reaches $nS\xi$ atoms. If it incides on the *total area of the lamina*, S_f , then the total number of atoms reached by the radiation is $N = nS_f\xi$. The number of atoms per unit of volume, n , is given by

$$n = \frac{N_0 \rho}{A} \quad (7)$$

where $N_0 = 6.02 \times 10^{26}$ atoms/kmole is the Avogadro's number; ρ is the matter density of the lamina (in kg/m^3) and A is the molar mass ($kg/kmole$).

When an electromagnetic wave incides on the lamina, it strikes N_f front atoms, where $N_f \cong (nS_f)\phi_m$, ϕ_m is the "diameter" of the atom. Thus, the electromagnetic wave incides effectively on an area $S = N_f S_m$, where $S_m = \frac{1}{4}\pi\phi_m^2$ is the cross section area of one atom. After these collisions, it carries out $n_{\text{collisions}}$ with the other atoms (See Fig.2).

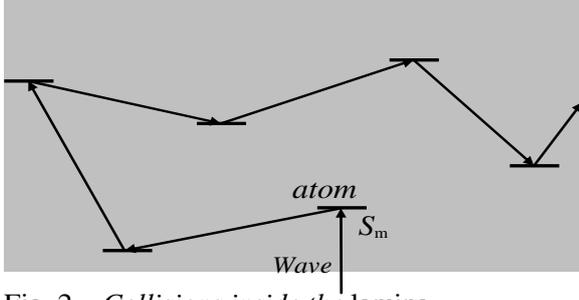


Fig. 2 – Collisions inside the lamina.

Thus, the total number of collisions in the volume $S\xi$ is

$$\begin{aligned} N_{\text{collisions}} &= N_f + n_{\text{collisions}} = n_f S \phi_m + (n_f S \xi - n_m S \phi_m) = \\ &= n_m S \xi \end{aligned} \quad (8)$$

The power density, D , of the radiation on the lamina can be expressed by

$$D = \frac{P}{S} = \frac{P}{N_f S_m} \quad (9)$$

We can express the *total mean number of collisions in each atom*, n_1 , by means of the following equation

$$n_1 = \frac{n_{\text{total photons}} N_{\text{collisions}}}{N} \quad (10)$$

Since in each collision a *momentum* h/λ is transferred to the atom, then the *total momentum* transferred to the lamina will be $\Delta p = (n_1 N) h/\lambda$. Therefore, in accordance with Eq. (1), we can write that

$$\begin{aligned} \frac{m_g(t)}{m_{i0}(t)} &= \left\{ 1 - 2 \left[\sqrt{1 + \left[(n_1 N) \frac{\lambda_0}{\lambda} \right]^2} - 1 \right] \right\} = \\ &= \left\{ 1 - 2 \left[\sqrt{1 + \left[n_{\text{total photons}} N_{\text{collisions}} \frac{\lambda_0}{\lambda} \right]^2} - 1 \right] \right\} \end{aligned} \quad (11)$$

Since Eq. (8) gives $N_{\text{collisions}} = n_f S \xi$, we get

$$n_{\text{total photons}} N_{\text{collisions}} = \left(\frac{P}{hf^2} \right) (n_f S \xi) \quad (12)$$

Substitution of Eq. (12) into Eq. (11) yields

$$\frac{m_g(t)}{m_{i0}(t)} = \left\{ 1 - 2 \left[\sqrt{1 + \left[\left(\frac{P}{hf^2} \right) (n_f S \xi) \frac{\lambda_0}{\lambda} \right]^2} - 1 \right] \right\} \quad (13)$$

Substitution of P given by Eq. (9) into Eq. (13) gives

$$\frac{m_g(t)}{m_{i0}(t)} = \left\{ 1 - 2 \left[\sqrt{1 + \left[\left(\frac{N_f S_m D}{f^2} \right) \left(\frac{n_f S \xi}{m_{i0}(t) c} \right) \frac{1}{\lambda} \right]^2} - 1 \right] \right\} \quad (14)$$

Substitution of $N_f \equiv (n_f S_f) \phi_m$ and $S = N_f S_m$ into Eq. (14) results

$$\frac{m_g(t)}{m_{i0}(t)} = \left\{ 1 - 2 \left[\sqrt{1 + \left[\left(\frac{n_f^3 S_f^2 S_m^2 \phi_m^2 D}{m_{i0}(t) c f^2} \right) \frac{1}{\lambda} \right]^2} - 1 \right] \right\} \quad (15)$$

where $m_{i0}(t) = \rho_{(t)} V_{(t)}$.

Now, considering that the lamina is inside an ELF electromagnetic field with E and B , then we can write that [8]

$$D = \frac{n_r(t) E^2}{2\mu_0 c} \quad (16)$$

Substitution of Eq. (16) into Eq. (15) gives

$$\frac{m_g(t)}{m_{i0}(t)} = \left\{ 1 - 2 \left[\sqrt{1 + \left[\left(\frac{n_r(t) n_f^3 S_f^2 S_m^2 \phi_m^2 E^2}{2\mu_0 m_{i0}(t) c^2 f^2} \right) \frac{1}{\lambda} \right]^2} - 1 \right] \right\} \quad (17)$$

In the case in which the area S_f is just the *area of the cross-section of the lamina* (S_α), we obtain from Eq. (17), considering that $m_{i0}(t) = \rho_{(t)} S_\alpha \xi$, the following expression

$$\frac{m_g(t)}{m_{i0}(t)} = \left\{ 1 - 2 \left[\sqrt{1 + \left[\left(\frac{n_r(t) n_f^3 S_\alpha S_m^2 \phi_m^2 E^2}{2\mu_0 \rho_{(t)} c^2 f^2} \right) \frac{1}{\lambda} \right]^2} - 1 \right] \right\} \quad (18)$$

If the electrical conductivity of the lamina, $\sigma_{(l)}$, is such that $\sigma_{(l)} \gg \omega \epsilon$, then the value of λ is given by Eq. (6), i.e.,

$$\lambda = \lambda_{\text{mod}} = \sqrt{\frac{4\pi}{\mu f \sigma}} \quad (19)$$

Substitution of Eq. (19) into Eq. (18) gives

$$\frac{m_g(l)}{m_{i0}(l)} = \left\{ 1 - 2 \left[\sqrt{1 + \frac{n_r^2 n_l^6 S_\alpha^2 S_m^4 \phi_m^4 \sigma_{(l)} E^4}{16\pi \mu_0 \rho_{(l)}^2 c^4 f^3}} - 1 \right] \right\} \quad (20)$$

Note that $E = E_m \sin \omega t$. The average value for E^2 is equal to $\frac{1}{2} E_m^2$ because E varies sinusoidally (E_m is the maximum value for E). On the other hand, $E_{\text{rms}} = E_m / \sqrt{2}$. Consequently we can change E^4 by E_{rms}^4 , and the equation above can be rewritten as follows

$$\chi = \frac{m_g(l)}{m_{i0}(l)} = \left\{ 1 - 2 \left[\sqrt{1 + \frac{n_r^2 n_l^6 S_\alpha^2 S_m^4 \phi_m^4 \sigma_{(l)} E_{\text{rms}}^4}{16\pi \mu_0 \rho_{(l)}^2 c^4 f^3}} - 1 \right] \right\} \quad (21)$$

Now consider the *Artificial Gravitational Lenses* shown in Fig.3.

Basically they are rectangular toroids. Inside them there are two *dielectric rings* with $\epsilon_r \cong 1$ and an *Aluminum ring* with mass density $\rho = 2700 \text{ kg.m}^{-3}$ (See Fig.3). The rectangular toroid is filled with *air* at ambient temperature and 1atm. Thus, inside the tube, the air density is

$$\rho_{\text{air}} \cong 1.2 \text{ kg.m}^{-3} \quad (22)$$

The number of atoms of air (Nitrogen) per unit of volume, n_{air} , according to Eq.(7), is given by

$$n_{\text{air}} = \frac{N_0 \rho_{\text{air}}}{A_N} = 5.16 \times 10^{25} \text{ atoms/m}^3 \quad (23)$$

Here, the area S_α refers to the area of the ring inside the air toroid, with average radius $\bar{r} = r_e + r_i / 2$ and height α , i.e.,

$$S_\alpha = 2\pi\alpha (r_i + r_e) / 2 = \pi\alpha (r_i + r_e)$$

where r_i is the inner radius and r_e the outer radius of the rectangular toroid. For $r_i = 400 \text{ mm}$, $r_e = 650 \text{ mm}$ and $\alpha = 60 \text{ mm}$, we get

$$S_\alpha = \pi\alpha (r_i + r_e) = 0.198 \text{ m}^2 \quad (24)$$

The *parallel metallic plates* (p), shown in Fig.3 are subjected to different drop voltages. The *two sets of plates* (D), placed on the extremes of the toroid, are subjected to $V_{(D)\text{rms}} = 0.441 \text{ V}$ at $f = 60 \text{ Hz}$, while the two sets of plates (A) are subjected to $V_{(A)\text{rms}} = 21.21 \text{ V}$ at $f = 60 \text{ Hz}$. Since $d = 98 \text{ mm}$, then the intensity of the electric field, which passes through the 36 *cylindrical air laminas* (each one with 5mm thickness) of the *two sets* (D), is

$$E_{(D)\text{rms}} = V_{(D)\text{rms}} / d = 4.5 \text{ V/m}$$

and the intensity of the electric field, which passes through the 14 *cylindrical air laminas* of the two sets (A), is given by

$$E_{(A)\text{rms}} = V_{(A)\text{rms}} / d = 216.43 \text{ V/m}$$

Note that the *metallic rings* (5mm thickness) are positioned in such way to block the electric field out of the cylindrical air laminas (also 5mm thickness). The objective is to turn each one of these laminas into a *Gravity Control Cell* (GCC) [6]. Thus, the system shown in Fig. 3 has 4 sets of GCC. Two with 18 GCC each and two with 7 GCC each. The two sets with 18 GCC each are positioned at the extremes of the tube (D). They work as gravitational *decelerator* while the other two set with 7 GCC (A) each works as a gravitational *accelerator*, intensifying the gravity acceleration produced by the *Aluminum ring*. According to Eq. (3), this

gravity after the 7th GCC becomes $g_7 = \chi^7 g_0$, where $\chi = m_{g(t)}/m_{i(t)}$ given by Eq. (21), and g_0 can be calculated starting from the expression of the gravitational mass of the *half-toroid* of Aluminum, $M_{g(\frac{1}{2}\text{toroid})}$, which is given by

$$\int_0^{M_g} dM_{g(\frac{1}{2}\text{toroid})} = \rho\alpha(r_e - r_i) \int_0^{\pi r_i} dz$$

whence

$$M_{g(\frac{1}{2}\text{toroid})} = \pi\rho\alpha(r_e - r_i)r_i \quad (25)$$

On the other hand, we have that

$$\int_0^g dg = -\frac{G}{r^2} \int_0^{M_g} dM_g = -\frac{G\alpha\rho(r_e - r_i)}{r^2} \int_0^{\pi r_i} dz$$

whence we get

$$g = -\frac{G\pi\alpha\rho(r_e - r_i)r_i}{r^2} \quad (26)$$

which gives the value of g produced by the half-toroid at a point inside the nucleus of the toroid, distant r from the center of the cross-section of the rectangular toroid. Thus, the value of g'_0 ($r = r_0$), due to the first half-toroid is

$$g'_0 \cong -G\pi\rho\alpha\left(\frac{r_e - r_i}{r_0^2}\right)r_i$$

The value of g''_0 , due to the opposite half-toroid is

$$g''_0 \cong -G\pi\rho\alpha\left\{\frac{(r_e - r_i)r_i}{\left[2r_e - \left(\frac{r_e - r_i}{2} + r_0\right) - \left(\frac{r_e - r_i}{2}\right)\right]^2}\right\}$$

Consequently, the resultant is

$$g_0 \cong -G\pi\rho\alpha\left(\frac{r_e - r_i}{r_0^2}\right)r_i - \left\{-G\pi\rho\alpha\left\{\frac{(r_e - r_i)r_i}{\left[2r_e - \left(\frac{r_e - r_i}{2} + r_0\right) - \left(\frac{r_e - r_i}{2}\right)\right]^2}\right\}\right\} = -G\pi\rho\alpha\left[\frac{(r_e - r_i)r_i}{(r_i + r_0)^2}\right]$$

In the case of $r_i \gg r_0$, the equation above reduces to

$$g_0 \cong -G\pi\rho\alpha\left(\frac{r_e - r_i}{r_0^2}\right)r_i \quad (27)$$

where r_i is the inner radius of the toroid; r_0 is the distance between the center of the cross-section of the Aluminum ring and the surface of the first GCC of the set (A); α is the thickness of the Aluminum ring. Here, $r_0 = 92.53\text{mm}$ and $\alpha = 60\text{mm}$ (See Fig. 3 (a)).

The objective of the sets (D), with 18 GCC each, is to reduce strongly the value of the external gravity along the *rectangular toroid of air* in D region. In this case, the value of the external gravity, g_{ext} , is reduced by the factor $\chi_d^{18} g_{ext}$, where $\chi_d = 10^{-2}$. For example, if $g_{ext} = 9.81\text{m/s}^2$ then this value is reduced to $\chi_d^{18} g_{ext}$ and, after the set A, it is increased by χ^7 . Since the system is designed for $\chi = -5128.2$, then the gravity acceleration on the Aluminum ring becomes $\chi^7 \chi_d^{18} g_{ext} = 9.15 \times 10^{-10} \text{m/s}^2$, this value is much smaller than $g_0 \cong -G\pi\rho\alpha\left[\frac{(r_e - r_i)r_i}{r_0^2}\right] = 9.9 \times 10^{-8} \text{m.s}^{-2}$.

Note that *Americium* 241[†] droplets are conveniently placed along the air toroids in order to increase the electrical conductivity of the air. The objective is to increase the conductivity of the air, inside the dielectric tube, up to $\sigma_{air} = 1 \times 10^{-6} S/m$. This value is of fundamental importance in order to obtain the convenient values of χ and χ_d , which are given by Eq. (21), i.e.,

$$\chi = \left\{ 1 - 2 \left[\sqrt{1 + \frac{n_{r(air)}^2 n_{air}^6 S_\alpha^2 S_m^4 \phi_m^4 \sigma_{air} E_{(A)rms}^4}{16\pi\mu_0 \rho_{air}^2 c^4 f^3}} - 1 \right] \right\} = \left\{ 1 - 2 \left[\sqrt{1 + 3 \times 10^{-3} E_{(A)rms}^4} - 1 \right] \right\} \quad (28)$$

$$\chi_d = \left\{ 1 - 2 \left[\sqrt{1 + \frac{n_{r(air)}^2 n_{air}^6 S_\alpha^2 S_m^4 \phi_m^4 \sigma_{air} E_{(D)rms}^4}{16\pi\mu_0 \rho_{air}^2 c^4 f^3}} - 1 \right] \right\} = \left\{ 1 - 2 \left[\sqrt{1 + 3 \times 10^{-3} E_{(D)rms}^4} - 1 \right] \right\} \quad (29)$$

where $n_{r(air)} = \sqrt{\mu_{r(air)} \sigma_{air} / 4\pi\epsilon_0 f} = 12.24$, $n_{air} = 5.16 \times 10^{25} \text{ atoms}/m^3$, $\phi_m = 1.55 \times 10^{-10} m$, $S_m = \pi\phi_m^2/4 = 1.88 \times 10^{-20} m^2$ and $f = 60 \text{ Hz}$. Since $E_{(A)rms} = 216.43 \text{ V}/m$ and $E_{(D)rms} = 4.5 \text{ V}/m$, we get

$$\chi = -5128.2 \quad (30)$$

and

$$\chi_d \cong 10^{-2} \quad (31)$$

Then the gravitational acceleration after the 7th gravitational shielding is

[†] The radioactive element *Americium* (Am-241) is widely used in *ionization smoke detectors*. This type of smoke detector is more common because it is inexpensive and better at detecting the smaller amounts of smoke produced by flaming fires. Inside an ionization detector there is a small amount (perhaps 1/5000th of a gram) of americium-241. The Americium is present in oxide form (AmO₂) in the detector. The cost of the AmO₂ is US\$ 1,500 per gram. The amount of radiation in a smoke detector is extremely small. It is also predominantly alpha radiation. Alpha radiation cannot penetrate a sheet of paper, and it is blocked by several centimeters of air. The americium in the smoke detector could only pose a danger if inhaled.

$$g_7 = \chi^7 g_0 = -\chi^7 G \pi \rho \alpha \left[\frac{(r_e - r_i) r_i}{r_0^2} \right] \quad (32)$$

It is known that gravitational fields can bend light, and that due to this effect, a light ray that passes very close to a body with gravitational mass M_g is deviated of an angle δ (deflection angle) given by [3]

$$\delta = -\frac{4GM_g}{c^2 d} \quad (33)$$

where d is the distance of closest approach.

Here, we can obtain the expression of δ as follows: by comparing Eq. (26) with Eq. (25) we obtain $GM_g = gr^2$. Substitution of this expression into Eq. (33) leads to the following equation

$$\delta = \frac{4gr^2}{c^2 d} \quad (34)$$

For $r = r_0$ we have $g = g_0$ and equation above can be rewritten as follows

$$\delta = \frac{4g_0 r_0^2}{c^2 d} \quad (35)$$

However, considering the symmetry of the gravitational lenses shown in Fig.3, it is easy to see that Eq. (35) must be rewritten as follows

$$\delta = \frac{4g_0 r_0^2}{c^2 d'} - \frac{4g_0 r_0^2}{c^2 d''} \quad (36)$$

where d' and d'' are respectively, the distances of closest approach of the light ray with respect to the two sides of the Aluminum ring (See Fig 3 (b)).

When the gravitational lenses are activated the value of g_0 is amplified to $\chi^7 g_0$, then Eq. (36) becomes

$$\delta = \frac{4\chi^7 g_0 r_0^2}{c^2 d'} - \frac{4\chi^7 g_0 r_0^2}{c^2 d''} \quad (37)$$

Note that, for $d' = d''$ (light ray *at the center* of the Gravitational lens) Eq. (37) gives $\delta = 0$ (*null deflection*). On the other hand, if $d' < d''$ we have $\delta > 0$ (the light ray is *gravitationally attracted* to the inner edge of rectangular toroid). Under these conditions, when a light flux crosses the gravitational lens (nucleus of the rectangular toroid), it becomes *divergent* in respect to the central axis of the toroid (See Fig. 3(c)). If $d' > d''$ then Eq. (37) shows that $\delta < 0$ (the light ray is *gravitationally repelled* from the inner edge of rectangular toroid). In this case, when a light flux crosses the gravitational lens, it becomes *convergent* in respect to the central axis of the toroid (See Fig. 3(b)).

Substitution of the known values into Eq. (37) yields

$$\delta = 0.135 \left(\frac{1}{d'} - \frac{1}{d''} \right) \quad (38)$$

Note that the values of δ can be easily controlled simply by controlling of the value of χ . Also note that the curvatures of the light rays are proportional to the distances d' and d'' , similarly to the curvature of the light rays in the *optical lenses*. Then it is easy to see that these gravitational lenses can be very useful in building of telescopes, microscopes, and in concentrating solar light in order to convert solar energy into thermal energy.

I would like to thank Physicist *André Luis Martins* (RJ, Brazil) who came up with the original idea to build Artificial Gravitational Lens using sets of Gravitational Shieldings, as shown in my previous papers.

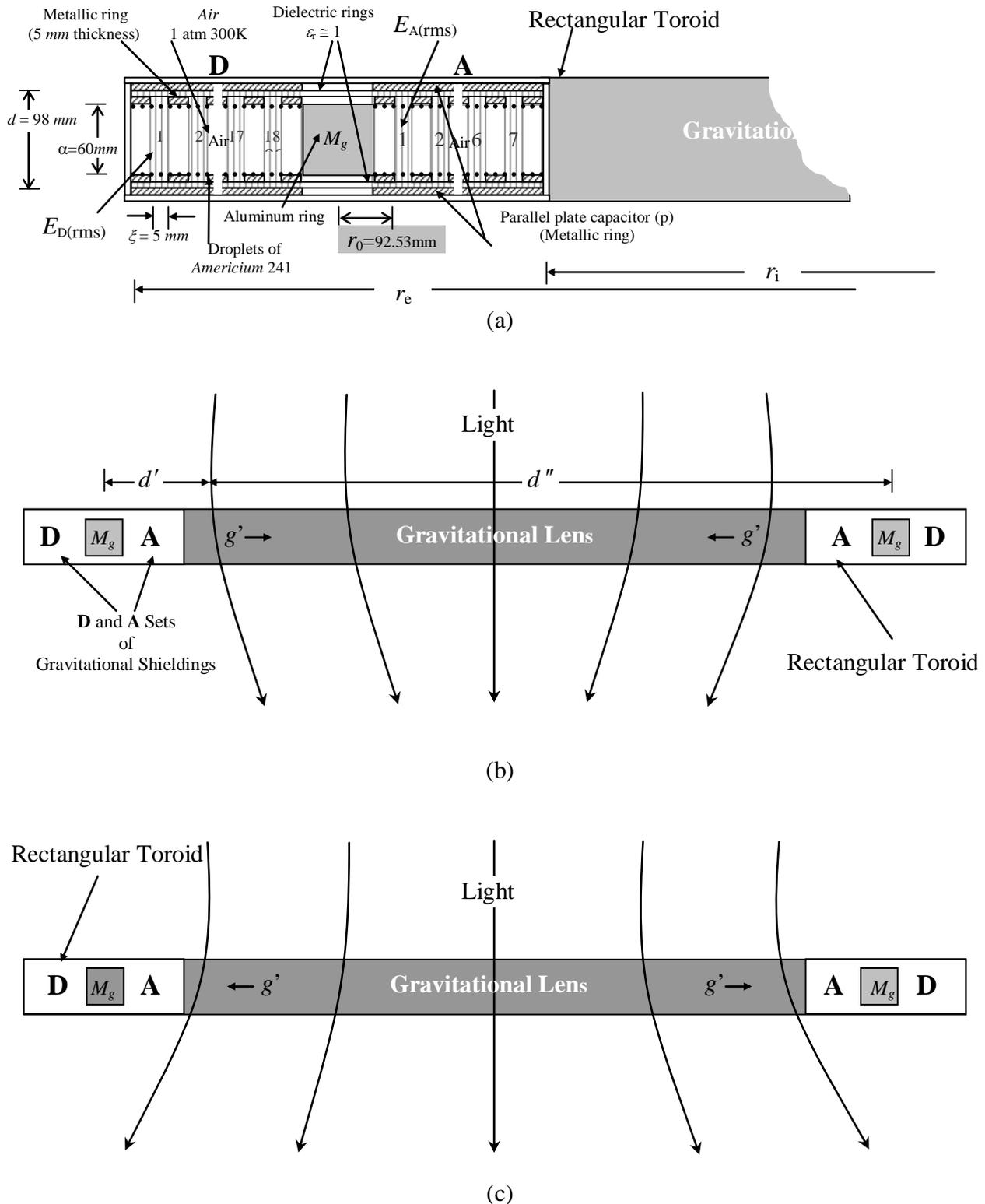


Fig. 3 – Artificial Gravitational Lens. (a) Cross-section of the Artificial Gravitational Lens. (b) Cross-section of a *Convergent* Gravitational Lens. The light rays are *gravitationally repelled* from the inner edge of toroid (c) Cross-section of a *Divergent* Gravitational Lens. The light rays are *gravitationally attracted* to the inner edge of toroid.

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